Pre-Calc Chapter 3 Final Review   Name________________________

1. Use the logistic function $y = \frac{60}{1+14e^{-.7x}}$ to answer the following questions:

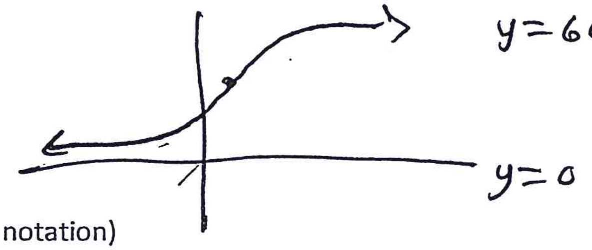
a. Find the y intercept

4

b. Use limit notation to describe the horizontal asymptotes

$x \to \infty$

$y \to 60$



c. Find the domain and range of the function (write answers in interval notation)

D: All Real ~~#s~~ $(-\infty, \infty)$

R: $(0, 60)$

d. Solve algebraically (or use "intersect") to find the value of x if y = 20

$x = \underline{2.7798}$

2nd Trace

$\boxed{5}$

Enter x 3

2. Write an exponential function with f(2) = 90 and f(5) = 60 (system of equations, no regression)

$y = a \cdot b^x$

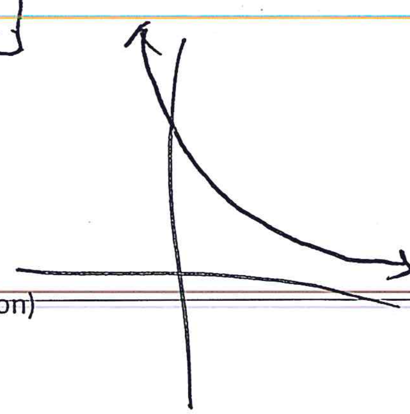
$$\frac{90 = a \cdot b^2}{60 = a b^5}$$

$(1.5)^{(-1/3)} = (b^{-3})^{(-1/3)}$

$\boxed{b = .87358..}$

$a = 117.933..$

$y = 117.933...(.87358..)^x$



2b. Find the domain and range of the function (write answers in interval notation)

D: $(-\infty, \infty)$

R: $(0, \infty)$

2c. Describe the end behavior of the function (fill in the blank below)

$\lim_{x\to\infty} f(x) =$ __0__

$\lim_{x\to-\infty} f(x) =$ __∞__

3. Write a Log Function with f(4) = 28 and f(6) = 40 (system of equations, no regression)

$\boxed{y = a + b \log x}$

$- \quad 28 = a + b \log 4$

$40 = a + b \log 6$

$12 = b \log 6 - b \log 4$

$12 = b(\log 6 - \log 4)$

$\boxed{b = 68.14}$

$a = -13.028...$

4. For each function, write an equation for the inverse function:

a. $f(x) = 4 - 9Logx$

b. $g(x) = 10 \cdot e^{3x}$

5. The table below shows the number of students who get a cold from other students at school in January:

Day	Number of Students with Colds
1	92
2	111
3	128
4	160
5	185
6	220
~~7~~	~~264~~

a. Use Logistic regression to write an equation that models the number of students with a cold

$y = \dfrac{1402.61...}{(1 + 17.37e^{-.195x})}$ Logistic L_1, L_2, Y_1

b. If the model follows, how many students will have a cold on day 20?

≈ 1040

c. According to the model, approximately how many students attend the school?

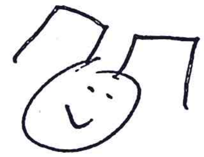
≈ 1403

6. Use regressions to write an equation for the exponential function below: → $y = 500.09(1.1199...)$

x	y
2	627
3	703
4	787
5	881
6	987

6b. Use your equation to find the value of x if y = 3000

15.82



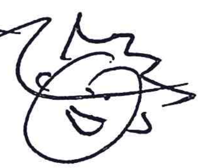
7. Use regressions to write an equation for the natural log function below:

x	y
1	15
2	20.5
3	23.8
4	26.1
5	27.9

$y = 14.98 + 8.02 \ln x$

7b. Use your equation to find the value of x if y =40

$x = 22.639604$



 +1 EC

Use the function $f(x) = Log_2 5x$ to answer the following questions (input graph into calculator):

$\frac{\log(5x)}{\log(2)}$

a. Find the domain and range of the function (write answers in interval notation)

D: $x > 0 \rightarrow (0, \infty)$ R: All Reals.

b. Find the x intercept of the function

$x = .2$

c. Use limit notation to describe the vertical asymptote of the function

$x \rightarrow 0^+$ $y \rightarrow -\infty$

d. Is the function increasing or decreasing? On what interval?

$(0, \infty)$

e. Is the function bounded? Explain briefly

No!

5. The half-life of Iodine-131 is 8.02 days.

$y = a\left(\frac{1}{2}\right)^{\left(\frac{x}{HL}\right)}$

a. Write a half-life equation for a 200 mg sample if Iodine-131

$y = 200\left(\frac{1}{2}\right)^{\left(\frac{x}{8.02}\right)}$

b. How many mg would remain after 4 weeks?

$\hookrightarrow 4 \times 7 = 28$

$y = 200\left(\frac{1}{2}\right)^{\left(\frac{28}{8.02}\right)} = 17.785$

c. How long will it take for there to be 10 mg remaining? (solve algebraically or use "intersect")

$x = 34.66$

$(0, 30)$ $(8, 25)$

6. A 30 gram sample of an unknown element has 25 grams remaining after 8 hours. Find the half-life of the element

$y = a(1/2)^{x/HL}$

$\cancel{30 = 30(1/2)^{0/HL}}$

$\boxed{a = 30}$

$25 = 30(1/2)^{8/HL}$

$\frac{25}{30} = \left(\frac{1}{2}\right)^{8/HL}$

$\log_{1/2}\left(\frac{25}{30}\right) = \frac{8}{HL}$

$HL \cdot .26303\ldots = \cancel{8}$

$\cancel{HL}$

$\boxed{HL = 30.41}$

7. For each function, write an inverse function:

a. $f(x) = 5e^{x+2}$

$y = 5e^{x+2}$

$x = 5e^{y+2}$

$\frac{x}{5} = e^{y+2}$

$\ln\left(\frac{x}{5}\right) = y + 2$

$\boxed{y = -2 + \ln\left(\frac{x}{5}\right)}$

b. $g(x) = 2Log(x-3)$ 10

$\frac{x}{2} = \frac{\cancel{2}\log_{10}(y-3)}{\cancel{2}}$

$\frac{x}{2} = \log_{10}(y-3)$

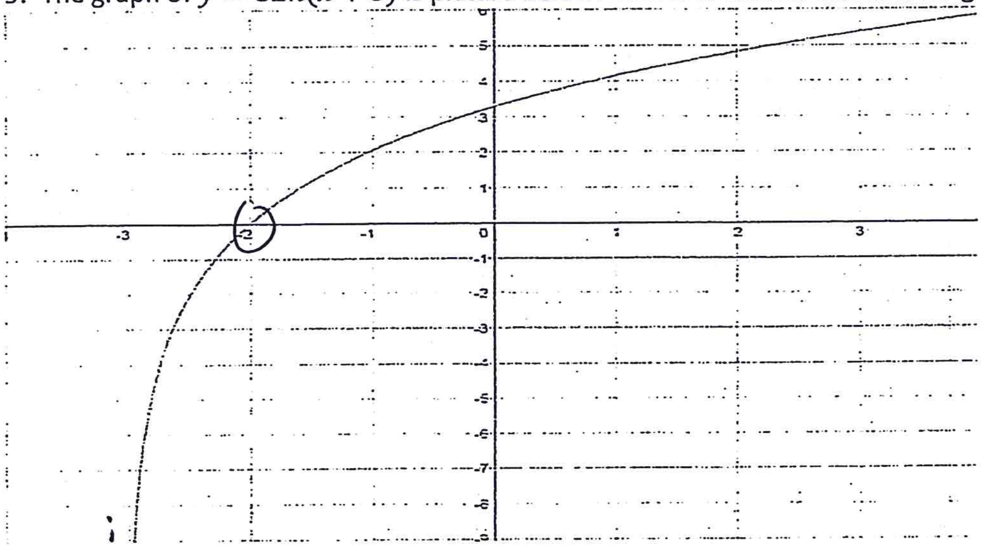
$10^{x/2} = y - 3$

$\boxed{10^{x/2} + 3 = y}$

8. Write a Log Function with f(4) = 18 and f(6)=28 (system of equations, no regression)

$y = a + b\log x$

9. The graph of $y = 3Ln(x+3)$ is picture below. Use it to answer the following questions



a. Write the domain and range of the function in interval notation

domain $(-3, \infty)$ range $\mathbb{R}$

b. is the function increasing or decreasing? INCREASINC c. Is it continuous? YEP!

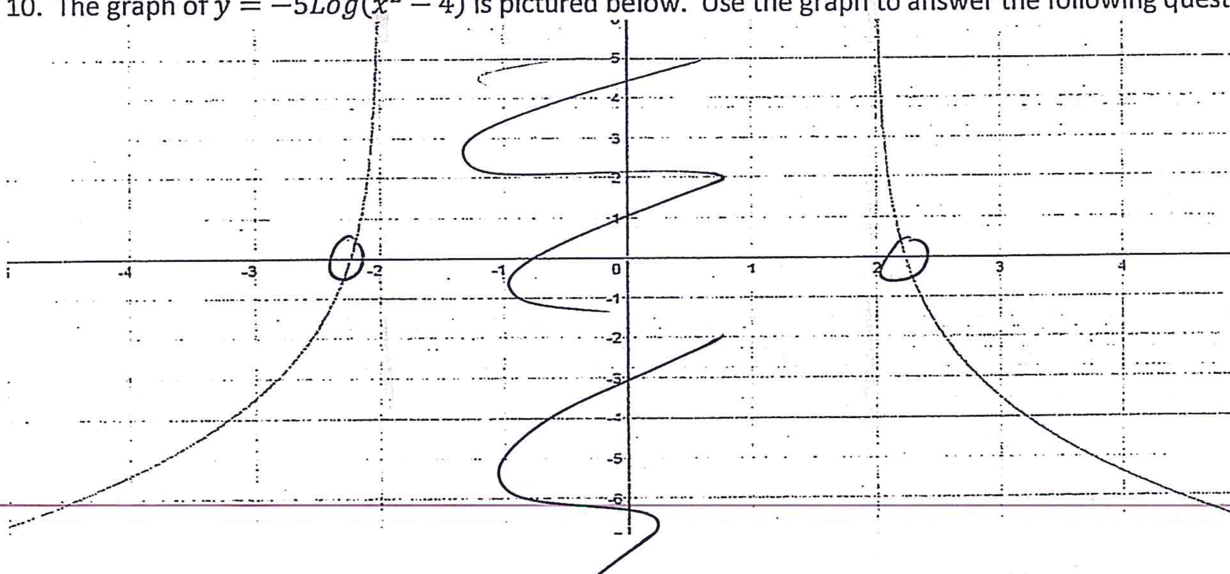
d. Use a limit to describe the asymptote $x \to -3^{+}$

$y \to -\infty$

e. Find the x and y intercept

$\boxed{-2}$ $\boxed{3.295\ldots}$

10. The graph of $y = -5Log(x^2 - 4)$ is pictured below. Use the graph to answer the following questions



a. Find the x intercepts of the function (exact values, do not estimate from the graph)

$10^0 = x^2$

$1 = x^2 - 4$

$\pm\sqrt{5}$

b. Write the domain of the function in interval notation $(-\infty, 2) \cup (2, \infty)$

c. Name ~~a~~ an interval on which the function is increasing and an interval on which the function is decreasing

11. You order a Starbucks coffee before school and bring it to first period. The temperature of the coffee when you set it on your desk is 150 degrees and the temperature in your first period classroom is 70 degrees. Your coffee cools to 100 degrees in 12 minutes. How long will it take for your coffee to cool to 75 degrees? (use newton's law of cooling formula, and follow the notes closely)

$$T(t) = T_m + (T_0 - T_m)e^{-kt}$$

$T_0 = 150°$

$T_m = 70°$

$(12, 100°)$

$$T(t) = \cancel{150}\ 70° + 80e^{-kt}$$

$$100° = \cancel{150}\ 70° + 80e^{-k(12)}$$

$$-\cancel{150}\ -70 \qquad -\cancel{150}\ -70$$

$$\frac{\cancel{-50}\ 30}{80} = e^{-k(12)}$$

$$\ln\left(\frac{\cancel{100}\ 30}{80}\right) = -k(12)$$

$$\frac{-.9808}{-12} = \frac{-k(12)}{-12}$$

$k = .081735...$

$$T(t) = 70 + 80e^{-.081735t}$$

$\boxed{x = 33.92 \text{ min.}}$

12. Solve the equation $e^{2x} + 20 = 9e^x$

$x = 1.609$

$y = 45$

13. Write an equation for an exponential function with f(3) = 10 and f(6) = 18 (system of equations, no regressions)

$y = a \cdot b^x$

$\cancel{18 \quad a\,b^6}$

$10 = a \cdot b^3$

$18 = a \cdot b^6$

$$\left(\frac{18}{10}\right)^{1/3} = \left(b^3\right)^{1/3}$$

$b = 1.21644.$

$a = 5.5550...$

Pre-Calc Chapter 2 Final Review Name______________________________

1. Given f(4) = 30 and f(6) = 40, write an equation for a power function

$y = ax^b$

$$\frac{40 = a(6)^b}{30 = a(4)^b}$$

$\frac{4}{3} = 1.5^b$

$\log_{1.5}(4/3) = b$

$b = \frac{\log(4/3)}{\log(1.5)}$ $\approx .71$

$\boxed{y = 11.21x^{.71}}$

$30 = a(4)^{.71}$

$30 = a(2.675...)$

$a = 11.21$

1b. Use your equation to find x if y = 70

$$\frac{70 = 11.21x^{.71}}{11.21 \quad 11.21}$$

$(6.24...)^{1/.71} = (x^{.71})^{1/.71}$

$\boxed{x = 13.195...}$

2. The data below models a power function:

Weight of Mammal (kg)	Pulse Rate (beats per minute)
0.3	300
2	205
5	120
30	85
50	70
70	72

a. Use regressions to write an equation for this power function

$y = 217.52x^{-.2774}$

b. Use your equation from part A to find the pulse rate of a mammal that weighs 20 kilograms

$x = 20$, $y = 94.75...$

c. Use your equation from part A to find the weight of a mammal with a pulse rate of 50 beats per minute

$50 = 217.52x^{-.2774}$

$\boxed{x = 200.37133}$

3. Let $g(x) = x^3 - 3x^2 - 3x + 2$ $x^3 + x^2 - 5x - 5$

a. List all of the possible rational zeros for the function (p/q)

$\frac{p}{q}$

$\pm 1, \pm 5$

b. Graph the function on the calculator and identify the 1 rational zero from the list above.

$x = -1$

c. Use your answer from part B and synthetic division to verify it's a zero of the function

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -5 & -5 \\ & \downarrow & -1 & 0 & 5 \\ \hline & 1 & 0 & -5 & \boxed{0} \end{array} \checkmark$$

d. Take the remaining quadratic from part C and use the quadratic formula to find the remaining 2 zeros

$1x^2 + 0x - 5 = 0$ $\qquad$ $x^2 = 5$

$x^2 - 5 = 0$ $\qquad$ $\boxed{x = \pm\sqrt{5}}$

$\sqrt{-100}$

4. The function $g(x) = x^2 - 4x + 29$ has no real solutions. Verify that x = 2 - 5i is a zero of the function $\sqrt{16-116}$

$$\begin{array}{r|rrr} 2-5i & 1 & -4 & 29 \\ & \downarrow & 2-5i & -29 \\ \hline & 1 & -2-5i & \boxed{0} \end{array}$$

OR

$x = \dfrac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 29}}{2 \cdot 1}$

$x = \dfrac{4 \pm 10i}{2}$

$\boxed{2 \pm 5i}$

5. Write the following polynomial function in standard form: $h(x) = (x+3)(x+4i)(x-4i)$

$\boxed{x^3 + 3x^2 + 16x + 48}$

$(x+3)(x^2+16)$

$x^3 + 16x + 3x^2 + 48$

6. The function $r(x) = x^3 - 8x^2 + 20x - 25$ has one real zero and 2 imaginary zeros

a. Graph the function on the calculator and find the 1 real zero.

$x = 5$

b. Use synthetic division with your answer from part A to reduce the function to a quadratic. Then take the remaining quadratic and use the quadratic formula to find the 2 imaginary zeros.

$$\begin{array}{r|rrrr} 5 & 1 & -8 & 20 & -25 \\ & \downarrow & 5 & -15 & 25 \\ \hline & 1 & -3 & 5 & \boxed{0} \end{array}$$

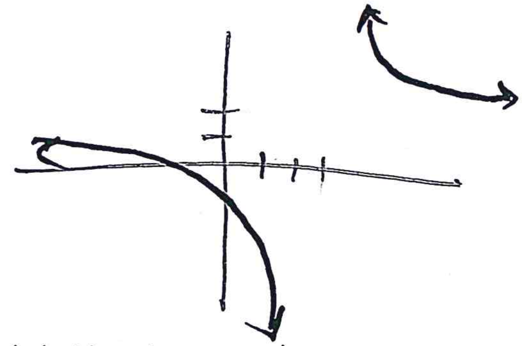
$x^2 - 3x + 5 = 0$

$x = \dfrac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot 5}}{2}$

$x = \dfrac{3 \pm \sqrt{-11}}{2}$

$x = \dfrac{3 \pm i\sqrt{11}}{2}$

7. Use the function $f(x) = \frac{2x+5}{x-3}$ to answer the following questions:



a. State the domain of the function in interval notation

$x \neq 3$ $\quad (-\infty, 3) \cup (3, \infty)$

b. Use limits to denote the vertical and horizontal asymptotes (graph could help but is not necessary)

Vertical $x = 3$

$\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 3^+} f(x) = \infty$

Horizontal $y = 2$

$\lim_{x \to -\infty} f(x) = 2$

$\lim_{x \to \infty} f(x) = 2$

c. Verify algebraically that y = 2 is in fact a horizontal asymptote

$(x-3)2 = \frac{2x+5}{\cancel{x-3}}(\cancel{x-3})$

$2x - 6 = 2x + 5$
$-2x \qquad -2x$

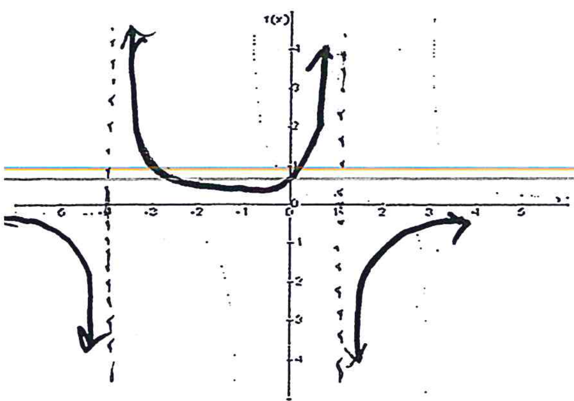
$\boxed{-6 = 5}$

NOT TRUE...

NO SOLUTION

8. Let $g(x) = \frac{x-3}{x^2+3x-4}$ (graph pictured below)

$\frac{(x-3)}{(x+4)(x-1)}$



~~a. Write the domain of the function in interval notation~~

$x \neq -4 \quad x \neq +1 \quad (-\infty, -4) \cup (-4, 1) \cup (1, \infty)$

b. Find the x and y intercept

y intercept

$x = 0$

$y = -.75$

x intercept

$x - 3 = 0$

$\boxed{x = 3}$

c. Use limits to denote any vertical asymptotes

$\lim_{x \to -4^-} f(x) = -\infty$

$\lim_{x \to -4^+} f(x) = \infty$

$\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = -\infty$

d. Use limits to denote the end behavior of the function

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = 0$

e. Verify numerically that for large value of x, $g(x) \approx 0$

$$\frac{(x-3)}{(x+4)(x-1)}$$

$x = 100$

$$\frac{(100-3)}{(100+4)(100-1)}$$

$$\frac{97}{(104)(99)} \approx \underline{.009}$$